Inseparability criteria for demonstration of the Einstein-Podolsky-Rosen gedanken experiment

M. D. Reid
Physics Department, University of Queensland, Brisbane, Australia
(February 1, 2008)

It is shown that a criterion used to demonstrate realization of the 1935 Einstein-Podolsky-Rosen (EPR) gedanken experiment is sufficient to demonstrate quantum entanglement. A further set of measurable criteria sufficient to demonstrate EPR gedanken experiment is proposed, these being the set of criteria sufficient to demonstrate entanglement, by way of a measured violation of a necessary condition of separability. In this way, provided the spatial separation of systems is sufficient to ensure EPR's locality hypothesis, it is shown how a measured demonstration of entanglement will, at least, be equivalent to a demonstration of the EPR gedanken experiment. Using hidden variables it is explained how such demonstrations are a direct manifestation of the inconsistency of local realism with quantum mechanics.

In 1935 Einstein, Podolsky and Rosen [1] (EPR) defined a premise called local realism. They showed that, for certain correlated spatially separated systems, if quantum mechanics is to be consistent with local realism, the position and momentum of a single particle must be simultaneously defined to a precision beyond the bounds given by the uncertainty principle. EPR took the view that local realism must be valid and therefore argued that quantum mechanics was incomplete.

Schrodinger's reply [2] in 1935 is now also well-known. In this reply Schrodinger introduced the concept of entangled quantum states, referring to their paradoxical nature for separated systems. Quantum entanglement is now a concept fundamental to all aspects of quantum information theory.

A criterion to demonstrate EPR correlations [3] for real experiments was presented in 1989. The first experimental achievement [4] of this EPR criterion, for measurements with continuous variable outcomes not impeded by detection inefficiencies, was presented by Ou et al in 1992. Here the two conjugate quantities are the quadrature phase amplitudes of the field [5], represented quantum mechanically by a quantized harmonic oscillator. There have now been further experimental observations [6,7] of continuous variable EPR correlations, and also proposals for further experiments [8]. While these EPR fields have proven significant in enabling the experimental realization of continuous variable quantum teleportation [9], and may have application also to quantum cryptography [10], the EPR experiments are also significant in providing a conclusive demonstration of the inconsistency of local realism with quantum mechanics, as we elaborate on in this paper.

My objective is to formalize the link between the original 1935 EPR argument and Schrodinger's 1935 essay introducing entanglement, by showing how the experimental demonstration of these EPR correlations will correspond to a demonstration of entanglement, and vice versa, provided measurements and spatial separations are sufficient to ensure the EPR locality hypothesis. This provides a means to demonstrate objectively the inconsistency of quantum entangled states with local realism.

In order to link EPR correlations with entanglement, I first consider a generalized EPR argument applying to situations of less than maximum correlation. It is then proven that the 1989 EPR criterion will always imply entanglement. It follows that continuous variable entanglement, an unconditional entanglement defined in the sense of Turchette et al [11], has been experimentally observed [4,6,7,12].

I next show that the local realistic description of a violation of certain sets of constraints, these being a set of necessary criteria for separability [13], necessitates a decomposition into local substates that, individually, defy the quantum bound set by the uncertainty relation. This is precisely the criterion for realization of an EPR gedanken experiment. It also follows from this logic that the entire quantum state predicting these violations must be inseparable, and in this way, with some further generalizations, we arrive at our result.

Crucial to demonstrating EPR correlations is the definition of local realism [1]. The premise of realism implies that if one can predict with certainty the result of a measurement of a physical quantity at a location A, without disturbing the system at A, then the results of the measurement were predetermined. There is an "element of reality" corresponding to this physical quantity, the element of reality being a variable that assumes one of the set of values that are the predicted results of the measurement. The locality assumption postulates that measurements at a spatially separated location B cannot immediately influence the subsystem at A.

EPR argued as follows. Consider two observables $\hat{x}$ and $\hat{p}$ for subsystem A, where $\hat{x}$ and $\hat{p}$ satisfy an uncertainty relation $\Delta\hat{x}\Delta\hat{p} \geq C$. Suppose one may predict with certainty the result of measurement $\hat{x}$ by a measurement performed at B, or alternatively, for a different measurement at B, the result of measurement $\hat{p}$. Assuming "local realism", we deduce the existence of an element of reality, $\tilde{x}$, for the physical quantity $\hat{x}$; and also an el-

ement of reality, $\tilde{p}$, for $\hat{p}$. Local realism then implies the existence of two hidden variables $\tilde{x}$ and $\tilde{p}$ that simultaneously determine, with no uncertainty, the values for the result of an $\hat{x}$ or $\hat{p}$ measurement on subsystem A, should it be performed. This hidden variable state for the subsystem A is not describable within quantum mechanics, because of the uncertainty relation.

For a link with inseparability, we extend the EPR argument to situations where the result of measurement $\hat{x}$ at A cannot be predicted with absolute certainty [3]. Local realism still allows us to deduce the existence of an element of reality ($\tilde{x}$) for $\hat{x}$ at A, since we can make a prediction of the result at A, without disturbing the subsystem at A, under the locality assumption. This prediction is based on the result y_i of a measurement $\hat{y}$ performed at B. The possible values for the "element of reality", the predicted results of the measurement $\hat{x}$, are no longer a set of definite numbers with zero uncertainty, but are a set of distributions, one for each possible result y_i for $\hat{y}$ at B. The element of reality $\tilde{x}$ becomes indeterminate, having a finite variance.

Each y_i represents a possible hidden variable state for the subsystem A. The predicted probability of a result for the measurement $\hat{x}$ at A, should the subsystem A be in the hidden variable state y_i, is given by the distribution labeled by y_i. This distribution is given formally by the conditional probability $P(x/y_i)$, the probability of obtaining a result x upon measurement of $\hat{x}$ at A, given the result y_i for the measurement of $\hat{y}$ at B. The probability that the subsystem A is in the hidden variable state designated y_i is $P(y_i)$, the probability of the result y_i at B, since through locality the action of measuring $\hat{y}$ at B could not have induced the result at A. We attribute to the inferred element of reality $\tilde{x}$, based on measurements $\hat{y}$, the weighted variance $\Delta^2_{inf,min}\hat{x} = \sum_{y_i} P(y_i){\Delta_i}^2$ where μ_i and Δ_i are the mean and standard deviation, respectively, of the conditional distribution $P(x/y_i)$.

The best estimate [14] of the outcome of $\hat{x}$ at A, based on a result y_i for the measurement at B, is given by μ_i, and $\Delta_i = \sqrt{\langle (x/y_i - \mu_i)^2\rangle}$ is the root mean square of the error or deviation $\delta_i = x/y_i - \mu_i$ in the inference μ_i. (Here x/y_i is the value obtained for x given the result y_i at B). $\Delta^2_{inf,min}\hat{x}$ defines the (minimum) average variance for the inference of the result of a measurement $\hat{x}$ at A, based on the result of the measurement $\hat{y}$ at B. Similarly for the inference of the result of measurement $\hat{p}$, based on a different measurement at B, we define a $\Delta_{inf,min}\hat{p}$.

We consider a measured error $\Delta_{inf}\hat{x}$ in the prediction for the outcome of measurement $\hat{x}$ at A, based on a result at B; and a similar measured error $\Delta_{inf}\hat{p}$ for the prediction of $\hat{p}$ at A. The 1989 criterion for demonstration of EPR correlations is to find

$$\Delta_{inf}\hat{x}\Delta_{inf}\hat{p} < C \tag{1}$$

since here elements of reality $\tilde{x}$, $\tilde{p}$ simultaneously attributed to system A by local realism are incompatible with the uncertainty principle.

We now show that the EPR criterion (1) is sufficient to demonstrate quantum entanglement. To do this we show that a separable quantum state, defined as expressible by a density matrix of the form

$$\rho = \sum_r P_r \rho_r^A \rho_r^B \tag{2}$$

where $\sum_r P_r = 1$, will imply $\Delta_{inf}\hat{x}\Delta_{inf}\hat{p} \geq C$. The conditional probability of result x for measurement $\hat{x}$ at A given a simultaneous measurement of $\hat{y}$ at B with result y_i is $P(x/y_i) = P(x, y_i)/P(y_i)$ where, given (2)

$$P(x, y_i) = \sum_r P_r P_r(y_i) P_r(x) \tag{3}$$

Here $|x\rangle, |y\rangle$ are the eigenstates of $\hat{x}$,$\hat{y}$ respectively, and $P_r(x) = \langle x|\rho_r^A|x\rangle$, $P_r(y_i) = \langle y_i|\rho_r^B|y_i\rangle$. The mean μ_i of this conditional distribution is $\mu_i = \sum_x xP(x/y_i) = \{\sum_r P_r P_r(y_i)\langle x\rangle_r\}/P(y_i)$ where $\langle x\rangle_r = \sum_x xP_r(x)$. The variance Δ_i^2 of the distribution $P(x/y_i)$ is $\Delta_i^2 = \{\sum_r P_r P_r(y_i)\sum_x P_r(x)(x-\mu_i)^2\}/P(y_i)$. For each state r, the mean square deviation $\sum_x P_r(x)(x-d)^2$ is minimized with the choice $d = \langle x\rangle_r$ [14]. Therefore for the choice $d = \mu_i$, $\Delta_i^2 \geq \{\sum_r P_r P_r(y_i)\sum_x P_r(x)(x - \langle x\rangle_r)^2\}/P(y_i) = \{\sum_r P_r P_r(y_i)\sigma_r^2(x)\}/P(y_i)$ where $\sigma_r^2(x)$ is the variance of $P_r(x)$. Taking the average variance over the y_i we get

$$\begin{aligned} \Delta^2_{inf}\hat{x} &\geq \sum_{y_i} P(y_i)\{\sum_r P_r P_r(y_i)\sigma_r^2(x)\}/P(y_i) \\ &= \sum_r P_r \sigma_r^2(x)\sum_{y_i} P_r(y_i) \\ &= \sum_r P_r \sigma_r^2(x) \end{aligned} \tag{4}$$

Also $\Delta^2_{inf}\hat{p} \geq \sum_r P_r\sigma_r^2(p)$, where $\sigma_r^2(p)$ is the variance of $P_r(p) = \langle p|\rho_r^A|p\rangle$, $|p\rangle$ being the eigenstate of $\hat{p}$. This implies (from the Cauchy-Schwarz inequality) $\Delta^2_{inf}\hat{x}\Delta^2_{inf}\hat{p} \geq \{\sum_r P_r\sigma_r^2(x)\}\{\sum_r P_r\sigma_r^2(p)\} \geq |\sum_r P_r\sigma_r(x)\sigma_r(p)|^2$. For any ρ_r^A it is constrained, by the uncertainty relation, that $\sigma_r(x)\sigma_r(p) \geq C$. We conclude that for a separable quantum state

$$\Delta_{inf}\hat{x}\Delta_{inf}\hat{p} \geq C. \tag{5}$$

The evaluation of the conditional probability distribution for each outcome y_i at B is not always be practical. We might propose the linear estimate $x_{est} = gy_i + d$ (g and d are constants) for the result x at A, given a result y_i for the measurement at B. The size of the deviation $\delta = x - (gy_i + d)$ can be measured. We simultaneously measure $\hat{x}$ at A and $\hat{y}$ at B, to determine x and y_i and then calculate for a given y_i, $\langle\delta^2\rangle_i = \sum_x P(x/y_i)\{x - (gy_i + d)\}^2$. Averaging over

the different values of y_i we obtain as a measure of error in our inference, based on the linear estimate: $\Delta^2_{inf,L}\hat{x} = \sum_{y_i} P(y_i)\langle\delta^2\rangle_i = \sum_{x,y_i} P(x,y_i)\{x-(gy_i+d)\}^2 = \langle\{\hat{x}-(g\hat{y}+d)\}^2\rangle$. The best linear estimate x_{est} is the one that will minimize $\Delta^2_{inf,L}\hat{x}$. This corresponds to the choice [14] $d = -\langle(\hat{x}-g\hat{y})\rangle$. (Denoting $\delta_0 = \hat{x}-g\hat{y}$, our choice of estimate optimized with respect to d gives a minimum error $\Delta^2_{inf,L}\hat{x} = \langle\delta_0^2 - \langle\delta_0\rangle^2\rangle$.) The best choice for g is discussed in [3]. The quantity $\Delta^2_{inf,L}\hat{x}$ may be measured straightforwardly, as discussed in [3] and [4,6,7].

If the estimate x_{est} corresponds to the mean of the conditional distribution $P(x/y_j)$ then the variance $\Delta^2_{inf,L}\hat{x}$ will correspond to the average conditional variance $\sum_{y_i} P(y_i){\Delta_i}^2$ specified above. This is the case, with a certain choice of g, for the two-mode squeezed state used to model continuous variable EPR states generated to date. In general the variances of type $\Delta^2_{inf,L}\hat{x}$ based on estimates will be greater than or equal to the optimal evaluated from the conditionals, and the separable quantum state must predict $\Delta_{inf,L}\hat{x}\Delta_{inf,L}\hat{p} \geq C$. The EPR criterion (1) so measured then implies inseparability, for any g and d. To show explicitly (optimizing d, but keep g general), separability implies (use [14])

$$\begin{aligned}
&\Delta^2_{inf,L}\hat{x} \geq \langle\{\hat{x}-\langle\hat{x}\rangle - g(\hat{y}-\langle\hat{y}\rangle)\}^2\rangle \\
&= \sum_{x,y}\sum_r P_r\langle x|\langle y|\rho^A_r\rho^B_r\{\hat{x}-\langle\hat{x}\rangle - g(\hat{y}-\langle\hat{y}\rangle)\}^2|x\rangle|y\rangle \\
&= \sum_r P_r\langle(\hat{\delta_0}-\langle\hat{\delta_0}\rangle)^2\rangle_r \geq \sum_r P_r\langle(\hat{\delta_0}-\langle\hat{\delta_0}\rangle_r)^2\rangle_r \qquad (6)
\end{aligned}$$

Here $\hat{\delta}_0 = \hat{x}-g\hat{y}$ and $\langle q\rangle_r$ denotes the average for state r given by density operator $\rho_r = \rho^A_r\rho^B_r$. Since ρ_r factorizes, $\langle\hat{x}\hat{y}\rangle_r = \langle\hat{x}\rangle_r\langle\hat{y}\rangle_r$. We have $\Delta^2_{inf,L}\hat{x} \geq \sum_r P_r(\langle\hat{\delta_0^2}\rangle_r - \langle\hat{\delta_0}\rangle^2_r) = \sum_r P_r(\Delta^2_r\hat{x} + g^2\Delta^2_r\hat{y})$ where $\Delta^2_r\hat{x} = \sigma^2_r(x)$ and $\Delta^2_r\hat{y} = \langle\hat{y}^2\rangle_r - \langle\hat{y}\rangle^2_r$. Also $\Delta^2_{inf,L}\hat{p} \geq \sum_r P_r(\Delta^2_r\hat{p} + h^2\Delta^2_r\hat{q})$ where $\Delta^2_r\hat{p} = \sigma^2_r(p)$ and $\hat{q}$ is the measurement at B used to infer the result for $\hat{p}$ at A. It follows (take $\Delta\hat{y}\Delta\hat{q} \geq D$) $\Delta^2_{inf,L}\hat{x}\Delta^2_{inf,L}\hat{p} \geq \sum_r P_r\{\sigma^2_r(x) + g^2\Delta^2_r\hat{y}\}\sum_r P_r\{\sigma^2_r(p) + h^2\Delta^2_r\hat{q}\}$. Separability implies

$$\Delta^2_{inf,L}\hat{x}\Delta^2_{inf,L}\hat{p} \geq (C^2 + g^2h^2D^2) \qquad (7)$$

I now propose a general method to demonstrate the EPR gedanken experiment, by which the incompatibility of the local realistic elements of reality with the uncertainty relation can be *inferred*, by way of violations of certain sets of constraints. We *propose*, without measurement, the existence of elements of reality, leaving *unspecified* the values for the variances of the elements of reality. At A one measures either x or p, a choice denoted by different values, 0 and π respectively, of a parameter θ. At B, simultaneously, there is the choice, denoted by ϕ, to measure an x_B or p_B. The set of elements of reality $\tilde{x}, \tilde{p}, \ldots$ for subsystem A, and $\tilde{x}_B, \tilde{p}_B, \ldots$ for subsystem B, form a set of hidden variables $\{\lambda\}$ for the entire system, with a probability distribution $\rho(\lambda)$. For each hidden variable state, there is a probability $p^A_x(\theta,\lambda)$ (independent of ϕ and with unspecified variance) for the result x of measurement θ at A. Similarly a $p^B_y(\phi,\lambda)$ is defined.

Assuming a general local hidden variable theory then, either as a consequence of EPR's local realism or as a new more general definition, the joint probability $P_{\theta,\phi}(x,y)$ of obtaining an outcome x at A and y at B is

$$P_{\theta,\phi}(x,y) = \int_\lambda \rho(\lambda) \quad p^A_x(\theta,\lambda)p^B_y(\phi,\lambda) \quad d\lambda \qquad (8)$$

Such local hidden variable theories were considered by Bell [15]. We also propose an auxiliary assumption, to now specify the variances of the elements of reality, by proposing that for each hidden variable state $\{\lambda\}$, the variances $\sigma^2_\lambda(x)$, $\sigma^2_\lambda(p)$ of $p^A_x(\theta=0,\lambda)$, $p^A_x(\theta=\pi,\lambda)$ respectively are restricted by the quantum "uncertainty principle" bound

$$\sigma_\lambda(x)\sigma_\lambda(p) \geq C \qquad (9)$$

Following the logic of (3) to (5), the local realistic theory (8) with assumption (9) will imply $\Delta_{inf}\hat{x}\Delta_{inf}\hat{p} \geq C$. In fact the general local realistic theory assumption (8), being separable in form, with the proviso (9) (and alternative provisos restricting the possible outcomes for a given hidden variable state to be within the domain predicted by a quantum state), will predict a whole set of inequalities or constraints, these being precisely the set of criteria derivable from the assumption of general quantum separability (2). For example from (7) above, quantum separability (2) implies (for any g) $\langle\{\hat{x}-\langle\hat{x}\rangle - g(\hat{y}-\langle\hat{y}\rangle)\}^2\rangle\langle\{\hat{p}-\langle\hat{p}\rangle - g(\hat{q}-\langle\hat{q}\rangle)\}^2\rangle \geq C^2(1+g^4)$, and this also follows from assumptions (8) and (9), as do the necessary conditions following from quantum separability where the uncertainty bound is used, derived in recent work by Duan et al [13], and Simon [13].

The demonstration of entanglement may be defined as the measured experimental violation of any one of the set of necessary criteria for separability (the results derivable from the general separable form (2)). Provided measurements and spatial separations between subsystems A and B allow justification of the locality assumption, such violations rule out, at least, the validity of all local hidden variables theories (8) where the hidden variable states (elements of reality) satisfy for each subsystem at A, and B, the bound (9) given by the uncertainty relation (or an alternative quantum bound). These violations are then none other than a demonstration of an EPR gedanken experiment, and lead to EPR's conclusion: that the predictions of quantum mechanics can only be represented by local realism, if the localized systems at A (B) that are necessarily part of the local realistic theory are described by something other than a quantum state satisfying the quantum bound (9). Quantum mechanics can

only be a local realistic theory, if it is "completed" to allow a violation of (9). In this way the inconsistency of quantum mechanics with local realism, a demonstration of the EPR paradox, is objectively demonstrable through entanglement criteria.

These more general entanglement criteria for demonstrating the EPR gedanken experiment are useful, since for example by (7) for $g = 1$ one needs only prove a two-mode squeezing result (of type $\Delta^2_{inf,L}\hat{x} < 2C^2$), averaged for both conjugate operators, to obtain $\Delta_{inf,L}\hat{x}\Delta_{inf,L}\hat{p} < 2C^2$; as opposed to searching for a $\Delta_{inf}\hat{x}\Delta inf\hat{p} < C^2$ with an optimal g.

Since the separable quantum state (2) satisfies the local hidden variable decomposition (8) with (9) satisfied ("completed" quantum states do not actually exist in quantum theory), it also follows from the very nature of the EPR argument that its demonstration can only come from quantum states that are inseparable.

A subset, these being the Bell-type inequalities [15], of the necessary criteria following from the local hidden variable decomposition (8) (and also from quantum separability (2)) do not require the additional assumption of a quantum bound (9). The conclusions drawn from such demonstrations of entanglement are stronger, in that all local realistic theories (8) are ruled out, even those "completing" quantum mechanics. Local realism itself is proved incorrect.

It has been shown possible in some cases to predict EPR correlations satisfying (1) from a local hidden variable theory, derived from the quantum Wigner function, that gives agreement with the quantum predictions for the direct x, p measurements. This implies that certain Bell inequalities for $\hat{x}, \hat{p}$ measurements will not be violated in this case. This might lead to the interpretation that the EPR experiment itself, demonstrating (1), reflected a situation in which quantum and local realistic (classical) domains are not distinguishable. This is not the case. The local realistic hidden variable theory used to give the quantum predictions is, necessarily, not actually quantum theory, since it must incorporate a description $\{\lambda_a\}$ for a state of the system at A or B in which the x and p are prespecified to a variance better than the uncertainty principle. The separable local hidden variable theory based on the Wigner function is not a separable (local) theory in quantum mechanics since these simultaneously well-defined x and p are not quantum states.

This is generally true where we have experimental violations of the necessary criteria for separability, whether viewed as a demonstration of entanglement or of EPR correlations; that it is proved that local realism can only be retained through certain theories alternative to quantum mechanics. In this way, the inconsistency of quantum mechanics with local realism is demonstrated through entanglement, it being the purpose of the subset Bell-type tests based only on (8) to rule out these further alternatives.

I am grateful to P. D. Drummond and H. J. Kimble for stimulating comments.

---

[1] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. **47**, 777, (1935).

[2] E. Schrödinger, Naturwissenschaften **23**, 807 (1935).

[3] M. D. Reid, Phys. Rev. A **40**, 913 (1989); Europhys. Lett. **36**, 1 (1996); Phys. Rev. Lett.**84**; Phys. Rev. A. **62** 022110 (2000); quant-ph/0101050.

[4] Z. Y. Ou, S. F. Pereira, H. J. Kimble and K. C. Peng, Phys. Rev. Lett. **68**, 3663 (1992).

[5] M. D. Reid and P. D. Drummond, Phys. Rev. Lett. 60, 2731, (1988).

[6] Yun Zhang, Hai Wang, Xiaoying Li,Jietai Jing, Changde Xie and Kunchi Peng, Phys. Rev. A **62**, 023813 (2000).

[7] Ch. Silberhorn, P. K. Lam, G. Wasik, N. Korolkova and G. Leuchs, presented at Europe IQEC (2000).

[8] V. Giovannetti, S. Mancini and P. Tombesi, quant-ph/0005066; K. Tara and G.S. Agarwal, Phys. Rev. A **50**, 2870, (1994); also E. Brambilla, A. Gatti, P. Navz and L. A. Lugiato, quant-ph/0010108.

[9] A. Furasawa et al, Science **282**, 706 (1998); L. Vaidmann, Phys. Rev. A **49**, 1473 (1994); S. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998); A. Kuzmich and E. S. Polzik, Phys. Rev. Lett. **85**, 5639 (2000); I. V. Sokolov, M. I. Kolobov, A. Gatti, L. A. Lugiato, quant-ph/0007026; P. Grangier and F. Grosshans, quant-ph/00010107.

[10] T. C. Ralph, Phys. Rev. A **61**, 303 (1999); M. Hillery, Phys. Rev. A **61**, 2309 (1999); M. D. Reid, Phys. Rev. A**62**, 062308 (2000); P. Navz, A. Gatti and G. Lugiato, quant-ph/0101113; also L. M. Duan, J. I. Cirac, P. Zoller, Phys. Rev. Lett. **85**, 5643 (2000); S. F. Pereira et al, quant-ph/0003094; Ch. Silberhorn et al, Europe IQEC (2000).

[11] Q. A. Turchette, C. S. Wood, B. E. King, C.J.Myatt, D. Liebfried, W. M. Itano, C. Monroe and D. J. Wineland, Phys. Rev. Lett. **81** , 3631 (1998).

[12] S. L. Braunstein et al, quant-ph/0012001.

[13] L. Duan, G. Giedke, J. I. Cirac and P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000); R. Simon, Phys. Rev. Lett. **84**, 2726 (2000); R. F. Werner and M. M. Wolf, quant-ph/0009118. (See also A. Peres, Phys. Rev. Lett. **77**, 1413,(1996)).

[14] This choice $d = \langle x \rangle$ will minimize $V = \langle (x-d)^2 \rangle$ and therefore the rms error (take $\partial V / \partial d$).

[15] J. S. Bell, "Speakable and Unspeakable in Quantum Mechanics" (Cambridge Univ. Press, Cambridge, 1988). A. Aspect, P. Grangier and G. Roger, Phys. Rev. Lett. **49**, 91, (1982).